# Different Approaches for Reward Shaping 

## Paniz Behboudian

Joint work with: Yash Satsangi, Matthew E. Taylor, Michael Bowling Summer 2019

## Outline

- Motivation
- Reward Shaping without Constraints
- Potential-Based Reward Shaping (PBRS)
- State Potentials
- State-Action Potentials
- Dynamic Potential-Based Reward Shaping
- Transforming any Signal into PBRS
- The Problem with Transforming any Signal into PBRS
- One Possible Solution


## Motivation



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## Markov Decision Process (MDP)

$$
\begin{aligned}
& M=\langle S, A, P, \gamma, R\rangle, \quad \pi(s): S \rightarrow A \\
& Q^{\pi}(s, a)=\mathbb{E}\left[\sum_{k=0}^{\infty} \gamma^{k} R\left(s_{t+k}, a_{t+k}\right) \mid s_{t}=s, a_{t}=a, \pi\right]
\end{aligned}
$$

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Q^{*}(s, a)=\max _{\pi \in \prod} Q^{\pi}(s, a), \quad \pi^{*}(s)=\arg \max _{a \in A} Q^{*}(s, a)
\end{gathered}
$$

## Shaping in Reinforcement Learning (RL)

$$
\begin{gathered}
R^{\prime}:=R+F \\
M=\langle S, A, P, \gamma, R\rangle \longrightarrow M^{\prime}=\left\langle S, A, P, \gamma, R^{\prime}\right\rangle
\end{gathered}
$$

## Shaping in Reinforcement Learning (RL)

Adding a reward in without constraint [1]:

$$
R^{\prime}:=R+F
$$



GOAL


1. Randløv, J., and Alstrøm, P. 1998. Learning to drive a bicycle using reinforcement learning and shaping

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## Potential-Based Reward Shaping (PBRS)

Constrain with PBRS [2]:

$$
R^{\prime}:=R+\overbrace{F}^{\gamma \Phi\left(s^{\prime}\right)-\Phi(s)}
$$

2. Ng, A. Y.; Harada, D.; and Russell, S. 1999. Policy invariance under reward transformations: Theory and application to reward shaping.

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GOAL

## Potential-Based Reward Shaping (PBRS)

Constrain with PBRS [2]:

-1-3
$-1-3$
$-1-3$


## Potential-Based Reward Shaping (PBRS)

Constrain with PBRS [2]:

$$
R^{\prime}:=R+
$$

$$
\gamma \overbrace{F} \cdot\left(s^{\prime}\right)-\Phi(s))
$$



$$
\Phi\left(s_{1}\right)=0 \quad \Phi\left(s_{2}\right)=3 \quad \Phi\left(s_{3}\right)=6 \quad \Phi\left(s_{4}\right)=9 \quad \Phi\left(s_{0}\right)=9
$$

## Potential-Based Reward Shaping (PBRS)

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\begin{array}{r}
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M=\langle S, A, P, \gamma, R\rangle \longrightarrow M^{\prime}=\left\langle S, A, P, \gamma, R^{\prime}\right\rangle \\
Q_{M}^{*}(s, a)=Q_{M^{\prime}}^{*}(s, a)+\Phi(s)
\end{array}
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Q_{M}^{*}(s, a)=Q_{M^{\prime}}^{*}(s, a)+\Phi \overbrace{\Phi(s)} \\
\text { Bias Term }
\end{array}
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\pi_{M}^{*} & =\pi_{M^{\prime}}^{*}
\end{aligned}
$$

## State-Action PBRS

Constrain with state-action PBRS [3]:

$$
\begin{gathered}
R^{\prime}:=R+\overbrace{F}^{\gamma \Phi\left(s^{\prime}, a^{\prime}\right)-\Phi(s, a)} \\
M=\langle S, A, P, \gamma, R\rangle \longrightarrow M^{\prime}=\left\langle S, A, P, \gamma, R^{\prime}\right\rangle \\
Q_{M}^{*}(s, a)=Q_{M^{\prime}}^{*}(s, a)+\Phi(s, a) \\
\pi_{M}^{*} \neq \pi_{M^{\prime}}^{*}
\end{gathered}
$$

3. Wiewiora, E.; Cottrell, G. W.; and Elkan, C. 2003. Principled methods for advising reinforcement learning agents.

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## State-Action PBRS

Constrain with state-action PBRS [3]:

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\begin{array}{r}
R^{\prime}:=R+\frac{\gamma \Phi\left(s^{\prime}, a^{\prime}\right)-\Phi(s, a)}{F} \\
M=\langle S, A, P, \gamma, R\rangle \longrightarrow M^{\prime}=\left\langle S, A, P, \gamma, R^{\prime}\right\rangle \\
Q_{M}^{*}(s, a)=Q_{M^{\prime}}^{*}(s, a)+\Phi(s, a) \\
\pi_{M^{\prime}}^{*}:=\operatorname{argmax}_{a}\left(Q_{M^{\prime}}^{*}(s, a)+\Phi(s, a)\right)
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## State-Action PBRS

## Constrain with state-action PBRS [3]:



Equivalent to state-action value initialization

$$
\begin{array}{r}
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\pi_{M^{\prime}}^{*}:=\operatorname{argmax}_{a}\left(Q_{M^{\prime}}^{*}(s, a)+\Phi(s, a)\right)
\end{array}
$$

## Dynamic PBRS

- Dynamic PBRS [4]:
- Used state-based dynamic PBRS in single and multiagent RL
- Proved the policy invariance
- Even before $\Phi$ stabilize


## Still need to define $\Phi$

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- Expressing any arbitrary rewards as potential-based advice [5]:
- Dynamic state-action shaping
- Learning $\Phi$ as a value function

5. Harutyunyan, A., Devlin, S., Vrancx, P., \& Nowé, A. (2015, February). Expressing arbitrary reward functions as potential-based advice.

## Expressing any arbitrary rewards as potential-based advice

$$
\begin{gathered}
R^{\prime}:=R+\stackrel{\gamma \Phi_{t+1}\left(s^{\prime}, a^{\prime}\right)-\Phi_{t}(s, a)}{F_{t}} \\
\Phi_{t+1}(s, a):=\Phi_{t}(s, a)+\beta \delta_{t}^{\Phi} \\
\delta_{t}^{\Phi}:=R^{\Phi}(s, a)+\gamma \Phi_{t+1}\left(s^{\prime}, a^{\prime}\right)-\Phi_{t}(s, a)
\end{gathered}
$$

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\Phi_{t+1}(s, a):=\Phi_{t}(s, a)+\beta \delta_{t}^{\Phi} \\
\delta_{t}^{\Phi}:=R^{R^{\Phi}(s, a)}+\gamma \Phi_{t+1}\left(s^{\prime}, a^{\prime}\right)-\Phi_{t}(s, a) \\
:=-R^{\text {expert }}
\end{gathered}
$$

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Q_{M}^{*}(s, a)=Q_{M^{\prime}}^{*}(s, a)+\Phi_{0}(s, a) \\
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Q_{M}^{*}(s, a)=Q_{M^{\prime}}^{*}(s, a)+\Phi_{\gamma}(s, a) \\
\pi_{M^{\prime}}^{*}:=\operatorname{argmax}_{a}\left(Q_{M^{\prime}}^{*}(s, a)+\Phi_{0}(s, a)\right)
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Q_{M}^{*}(s, a) & =Q_{M^{\prime}}^{*}(s, a)+\Phi_{\gamma}(s, a) \\
\pi_{M}^{*} & =\pi_{M^{\prime}}^{*}
\end{aligned}
$$

## Experiments

$$
R\left(s, s^{\prime}\right):= \begin{cases}1 & \text { if } s^{\prime}=G \\ 0 & \text { o.w. }\end{cases}
$$



$$
R_{\text {expert }}\left(s, s^{\prime}\right):=-\| \text { next_state }(s)-s^{\prime} \|
$$

## Experiments

$$
\begin{aligned}
& R\left(s, s^{\prime}\right):= \begin{cases}1 & \text { if } s^{\prime}=G \\
0 & \text { o.w. }\end{cases} \\
& R_{\text {expert }}\left(s, s^{\prime}\right):=-| | \text { next_state }(s)-s^{\prime}| |
\end{aligned}
$$

# Experiments: Dynamic PBRS 

Sarsa(0), $\gamma=0.3, \epsilon$-greedy policy



## Expressing any arbitrary rewards as potential-based advice

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\begin{array}{r}
R^{\prime}:=R+\frac{\gamma \Phi_{t+1}\left(s^{\prime}, a^{\prime}\right)-\Phi_{t}(s, a)}{F_{t}} \\
M=\langle S, A, P, \gamma, R\rangle \longrightarrow M^{\prime}=\left\langle S, A, P, \gamma, R^{\prime}\right\rangle \\
Q_{M}^{*}(s, a)=Q_{M^{\prime}}^{*}(s, a)+\Phi_{0}(s, a) \\
\pi_{M^{\prime}}^{*}:=\operatorname{argmax}_{a}\left(Q_{M^{\prime}}^{*}(s, a)+\Phi_{0}(s, a)\right)
\end{array}
$$

## The wrong bias

$$
\begin{array}{r}
R^{\prime}:=R+\frac{\gamma \Phi_{t+1}\left(s^{\prime}, a^{\prime}\right)-\Phi_{t}(s, a)}{F_{t}} \\
M=\langle S, A, P, \gamma, R\rangle \longrightarrow M^{\prime}=\left\langle S, A, P, \gamma, R^{\prime}\right\rangle \\
Q_{M}^{*}(s, a)=Q_{M^{\prime}}^{*}(s, a)+\Phi_{0}(s, a) \\
\pi_{M^{\prime}}^{*}:=\operatorname{argmax}_{a}\left(Q_{M^{\prime}}^{*}(s, a)+\Phi_{0}(s, a)\right.
\end{array}
$$

## The corrected bias

$$
\begin{gathered}
R^{\prime}:=R+\frac{\gamma \Phi_{t+1}\left(s^{\prime}, a^{\prime}\right)-\Phi_{t}(s, a)}{F_{t}} \\
M=\langle S, A, P, \gamma, R\rangle \longrightarrow M^{\prime}=\left\langle S, A, P, \gamma, R^{\prime}\right\rangle \\
Q_{M}^{*}(s, a)=Q_{M^{\prime}}^{*}(s, a)+\Phi_{t}(s, a) \\
\pi_{M^{\prime}}^{*}:=\operatorname{argmax}_{a}\left(Q_{M^{\prime}}^{*}(s, a)+\Phi_{t}(s, a)\right)
\end{gathered}
$$

## Experiments

Sarsa(0), $\gamma=0.3, \epsilon$-greedy policy


## Experiments: Corrected Dynamic PBRS

Sarsa(0), $\gamma=0.3, \epsilon$-greedy policy



## Experiments

Sarsa(0), $\gamma=0.3, \epsilon$-greedy policy


## Experiments

Sarsa(0), $\gamma=0.3, \epsilon$-greedy policy


## Experiments: Corrected Dynamic PBRS

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## Experiments: Corrected Dynamic PBRS

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## Another Look at Shaped Values

$$
\begin{gathered}
R^{\prime}:=R+\frac{\gamma \Phi_{t+1}\left(s^{\prime}, a^{\prime}\right)-\Phi_{t}(s, a)}{F_{t}} \\
M=\langle S, A, P, \gamma, R\rangle \longrightarrow M^{\prime}=\left\langle S, A, P, \gamma, R^{\prime}\right\rangle \\
Q_{M}^{*}(s, a)=Q_{M^{\prime}}^{*}(s, a)+\Phi_{t}(s, a) \\
\pi_{M^{*}}^{*}:=\operatorname{argmax}_{a}\left(Q_{M^{*}}^{*}(s, a)+\Phi_{t}(s, a)\right)
\end{gathered}
$$

## Another Look at Shaped Values

$$
\begin{gathered}
Q_{M}^{*}(s, a)=Q_{M^{\prime}}^{*}(s, a)+\Phi_{t}(s, a) \\
\pi_{M^{\prime}}^{*}=\operatorname{argmax}_{a}\left(Q_{M^{\prime}}^{*}(s, a)+\eta \Phi_{t}(s, a)\right)
\end{gathered}
$$

## Another Look at Shaped Values

$$
\begin{gathered}
Q_{M}^{*}(s, a)=Q_{M^{\prime}}^{*}(s, a)+\Phi_{t}(s, a) \\
\pi_{M^{\prime}}^{*}=\operatorname{argmax}_{a}\left(Q_{M^{\prime}}^{*}(s, a) \pm \eta \Phi_{t}(s, a)\right) \\
\text { Starts from } 0 \text { and reaches } 1
\end{gathered}
$$

## Another Look at Shaped Values

$$
\begin{gathered}
Q_{M}^{*}(s, a)=Q_{M^{\prime}}^{*}(s, a)+\Phi_{t}(s, a) \\
\pi_{M^{\prime}}^{*}=\operatorname{argmax}_{a}\left(Q_{M}^{*}(s, a)-\Phi_{t}(s, a)+\eta \Phi_{t}(s, a)\right) \\
\pi_{M^{\prime}}^{*}=\operatorname{argmax}_{a}\left(Q_{M}^{*}(s, a)-(1-\eta) \Phi_{t}(s, a)\right)
\end{gathered}
$$

## Another Look at Shaped Values

$$
\begin{gathered}
Q_{M}^{*}(s, a)=Q_{M^{\prime}}^{*}(s, a)+\Phi_{t}(s, a) \\
\pi_{M^{\prime}}^{*}=\operatorname{argmax}_{a}\left(Q_{M}^{*}(s, a)-\Phi_{t}(s, a)+\eta \Phi_{t}(s, a)\right) \\
\pi_{M^{\prime}}^{*}=\operatorname{argmax}_{a}\left(Q_{M}^{*}(s, a)-\xi \Phi_{t}(s, a)\right)
\end{gathered}
$$

## Soft-Shaped

$$
\begin{gathered}
Q_{M}^{*}(s, a)=Q_{M^{\prime}}^{*}(s, a)+\Phi_{t}(s, a) \\
\pi_{M^{\prime}}^{*}=\operatorname{argmax}_{a}\left(Q_{M}^{*}(s, a)-\Phi_{t}(s, a)+\eta \Phi_{t}(s, a)\right) \\
\pi_{M^{\prime}}^{*}=\operatorname{argmax}_{a}\left(Q_{M}^{*}(s, a)-\xi \Phi_{t}(s, a)\right) \\
\text { Starts from } 1 \text { and reaches } 0
\end{gathered}
$$

## Experiments: Soft-Shaped



## Summary

-A brief overview of the RL shaping literature

- Pointing out the necessary correction of bias term in dynamic PBRS framework
- Supporting Experiments
-A possible solution


## Thanks for attending!

